The Concept of a Stable Model: Early History and Some Recent Developments

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1976: Maarten van Emden and Robert Kowalski, *The semantics of predicate logic as a programming language*.

1978: Keith Clark, *Negation as failure*.

1988: Krzysztof Apt, Howard Blair and Adrian Walker, *Towards a theory of declarative knowledge*.

1988: Allen Van Gelder, *Negation as failure using tight derivations for general logic programs*.

1980: John McCarthy, *Circumscription—a form of non-monotonic reasoning*.

1980: Raymond Reiter, *A logic for default reasoning*.

1982: Raymond Reiter, *Circumscription implies predicate completion (sometimes)*.

1985: Robert Moore, *Semantical considerations on nonmonotonic logic*.

1987: Michael Gelfond, *On stratified autoepistemic theories*.

$$r(X) \leftarrow p(X), not\ q(X).$$

$$r(a) \leftarrow p(a), not\ q(a).$$
$$r(b) \leftarrow p(b), not\ q(b).$$

$$p(a) \wedge \neg \mathsf{L}\, q(a) \rightarrow r(a),$$
$$p(b) \wedge \neg \mathsf{L}\, q(b) \rightarrow r(b).$$

1987: Nicole Bidoit and Christine Froidevaux, *Minimalism subsumes default logic and circumscription in stratified logic programming*.

$$r(X) \leftarrow p(X), not\ q(X).$$

$$\frac{p(X)\ :\ \mathsf{M}\,\neg q(X)}{r(X)}.$$

1988: Michael Gelfond and Vladimir Lifschitz, *The stable model semantics for logic programming*.

Reduct relative to M:

(i) drop each rule $A_0 \leftarrow A_1, \ldots, A_m, not\ A_{m+1}, not\ A_n$ containing a term $not\ A_i$ with $A_i \in M$, and

(ii) drop all terms $not\ A_i$ from the remaining rules.

1989: Kit Fine, *The justification of negation as failure*.

1990: Michael Gelfond and Vladimir Lifschitz, *Logic programs with classical negation*.

1997: David Pearce, *A new logical characterization of stable models and answer sets*.

$$\frac{\text{equilibrium logic}}{\text{circumscription}} = \frac{\text{Kripke models}}{\text{classical models}}$$

This characterization

- treats ground rules as shorthand for propositional formulas: $r(a) \leftarrow p(a), not\ q(a)$ is identified with $p(a) \wedge \neg q(a) \rightarrow r(a)$;
- refers to Kripke models, but not to reducts;
- does not emphasize the role of negation: $\neg A$ can be viewed as shorthand for $A \rightarrow \bot$.

ASP Constructs	Propositional formulas
$q\,;r \leftarrow p$	$p \rightarrow q \vee r$
$\{q, r\} \leftarrow p$	$p \rightarrow (q \vee \neg q) \wedge (r \vee \neg r)$
$\leftarrow p, not\ q$	$\neg(p \wedge \neg q)$
$s \leftarrow 1\{p, q, r\}$	$p \vee q \vee r \rightarrow s$

Traditional definition of the reduct:

(i) drop each rule $A_0 \leftarrow A_1, \ldots, A_m, not\ A_{m+1}, not\ A_n$ containing a term $not\ A_i$ with $A_i \in M$, and

(ii) drop all terms $not\ A_i$ from the remaining rules.

2004: Wolfgang Faber, Nicola Leone and Gerald Pfeifer, *Recursive aggregates in disjunctive logic programs: semantics and complexity*.

drop each rule $A_0 \leftarrow A_1, \ldots, A_m, not\ A_{m+1}, not\ A_n$ containing a term A_i such that $A_i \notin M$ or a term $not\ A_i$ such $A_i \in M$.

2005: Paolo Ferraris, *Answer sets for propositional theories*.

The reduct of a propositional formula F relative to M is obtained from F by replacing every maximal subformula of F that is not satisfied by M with $\perp$.

2007: Paolo Ferraris, Joohyung Lee and Vladimir Lifschitz, *A new perspective on stable models*.

Stable models of a first-order formula F are defined as the models of F that satisfy a certain condition expressed in second-order logic.

2008: Joohyung Lee, Vladimir Lifschitz and Ravi Palla, *A reductive semantics for counting and choice in answer set programming*.

The semantics of RASPL-1 translates the rule

$$\{p(X)\} \leftarrow q(X), \{Y : r(X,Y)\}\, 1$$

into the formula

$$\forall X [q(X) \wedge \neg\exists Y_1 Y_2 (r(X,Y_1) \wedge q(X,Y_2) \wedge Y_1 \neq Y_2) \\ \rightarrow p(X) \vee \neg p(X)].$$

Conclusion

The invention of stable models was motivated by desire to "justify" the use of negation in logic programming.

Later on, this work led to the emergence of a valuable knowledge representation language.

From this perspective, the initial emphasis on the problem of negation appears to be a historical accident.